Supporting Students with Learning Disabilities in Mathematics

Individual	**I**	Inclusion
Education	**E**	Engagement
Plan	**P**	Performance

A YCDSB Resource to Support Implementation

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(Revised February 2015)

Patricia Preston
Director of Education

Elizabeth Crowe
Chair

Our Context

Across the province of Ontario and within the York Catholic DSB, students with special education needs are struggling in mathematics. As teachers of mathematics and special education, we are called to respond to this *opportunity*.

This document, which focuses on supporting students with learning disabilities, serves as a tool to develop, interpret, and implement Individualized Education Plans (IEPs) in mathematics. It is intended to be used collaboratively by teachers of mathematics and special education with the vision of reducing the gap in student achievement in mathematics between the general student population and students with learning disabilities, within schools, the board and the province.

Through the use of this document, we anticipate that mathematics and special education teachers will deepen their understanding of individual student learning profiles and respond with precision and personalization to student learning needs in mathematics. Together teachers of mathematics and special education will support meaningful *inclusion*, *engagement,* and *performance* of students with learning disabilities in the mathematics classroom.

What We Know

- In 2011, in the province of Ontario, approximately 14.5% of students received special education programs and services. 85% of these students are served within a regular classroom setting. [1]

- Close to half of students who receive special education services have a learning disability. [1]

- Over the past fifteen years, various sources of student achievement data (e.g. EQAO, report card data, credit acquisition) indicate that students with a learning disability are struggling in mathematics. In the York Catholic DSB, 47% of students with a learning disability who were enrolled in Grade 9 Applied mathematics achieved L3 or L4 on the 2013-14 EQAO assessment. Significant gains have been made over the past four years (+13%), and yet our work is just beginning.

- **Students with a learning disability have average to above average cognitive ability. They can learn mathematics.**

What We Believe

- Students must have a solid conceptual foundation in mathematics in order to apply their knowledge and to continue to learn mathematics. [2]

- Effective instructional strategies in mathematics emphasize the ability to think, to solve problems, and to build one's own understandings. [2]

> All students can learn mathematics, and they deserve the opportunity to do so… However, recognizing the diversity among … children, educators do not expect all students to learn the material in the same manner, using the same resources, and in the same time frame."
> *(Sutton & Krueger, 2002, p.1)*

- Classroom teachers are the key educators for students' … numeracy development. [3]

- Classroom teachers [of mathematics] need the support of the larger community to create a learning environment that supports students with special education needs. [3]

- Through the collaborative efforts of mathematics and special education teachers, all students with learning disabilities can succeed in mathematics

The Intended Use Of This Resource Is To:

- Support teachers in developing, interpreting, and implementing Individualized Education Plans (IEPs) in mathematics.
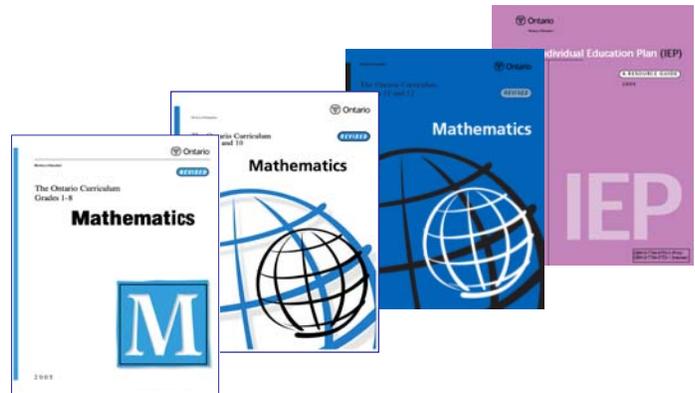
- Develop understanding of the cognitive domains / processes involved in learning and how each of these impacts on student engagement in the mathematical processes, otherwise known as the *actions of mathematics*.
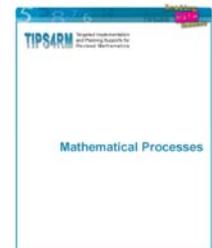
- Support teachers of mathematics and special education in working collaboratively to select, with precision and personalization, appropriate accommodations based on the unique profile of a student with a learning disability (ie. strengths and needs, as outlined in the IEP).

  Breakthrough by Fullan, Hill, Crevola (2006)

- Maximize student learning and opportunities to demonstrate learning in mathematics.

The Mathematical Processes

Presented at the start of every course in this curriculum document is a set of seven expectations that describe the mathematical processes students need to learn and apply as they work to achieve the expectations outlined within the five strands / strands of the course. [4]

The mathematical processes that support effective learning in mathematics are as follows:

Problem solving
Students will develop, select, apply, and compare a variety of problem-solving strategies as they pose and solve problems and conduct investigations, to help deepen their mathematical understanding. [5]

Reasoning and Proving
Students will develop and apply reasoning skills (e.g., recognition of relationships, generalization through inductive reasoning, use of counter-examples) to make mathematical conjectures, assess conjectures and justify conclusions, and plan and construct organized mathematical arguments. [5]

Reflecting
Students will demonstrate that they are reflecting on and monitoring their thinking to help clarify their understanding as they complete an investigation or solve a problem (e.g., by assessing the effectiveness of strategies and processes used, by proposing alternative approaches, by judging the reasonableness of results, by verifying solutions). [5]

Selecting Tools and Computational Strategies
Students will select and use a variety of concrete, visual, and electronic learning tools, and appropriate computational strategies to investigate mathematical ideas and solve problems. [5]

Connecting
Students will make connections among mathematical concepts and procedures, and relate mathematical ideas to situations or phenomena drawn from other contexts (e.g., other curriculum areas, daily life, current events, art and culture, sports). [5]

Representing
Students will create a variety of representations of mathematical ideas (e.g., numeric, geometric, algebraic, graphical, pictorial; onscreen dynamic representations), connect and compare representations, and select and apply the appropriate representations to solve problems. [5]

Communicating
Students will communicate mathematical thinking orally, visually, and in writing, using mathematical vocabulary and a variety of appropriate representations, and observing mathematical conventions. [5]

The mathematical processes are interconnected. Problem solving and communicating have strong links to all the other processes. [6]

The mathematical processes cannot be separated from the knowledge and skills that students acquire throughout the year / course. Students must problem solve, communicate, reason, reflect, and so on, as they develop the knowledge, the understanding of concepts, and the skills required in all the strands in every grade / course. [6]

Roles and Responsibilities in Mathematics Programs

It is important for teachers to use a variety of instructional, assessment, and evaluation strategies, in order to provide numerous opportunities for students to develop their ability to solve problems, reason mathematically, and connect the mathematics they are learning to the real world around them. Opportunities to relate knowledge and skills to wider context will motivate students to learn and to become lifelong learners. [7]

Teaching Approaches

To make new learning more accessible to students, teachers draw upon the knowledge and skills students have acquired in previous years – in other words, they help activate prior knowledge. It is important to assess where students are in their mathematical growth and to bring them forward in their learning. [8]

In order to apply their knowledge effectively and to continue to learn, students must have a solid conceptual foundation in mathematics. Successful classroom practices involve students in activities that require higher-order thinking, with an emphasis on problem solving. Students who have completed the elementary program should have a good grounding in the investigative approach to learning new concepts, including the inquiry model of problem solving, and this approach is still fundamental in [secondary mathematics courses]. [8]
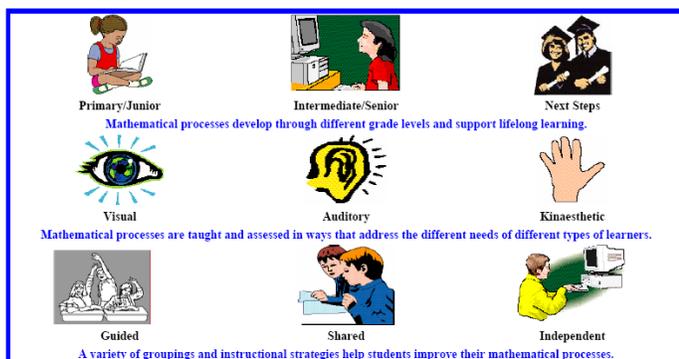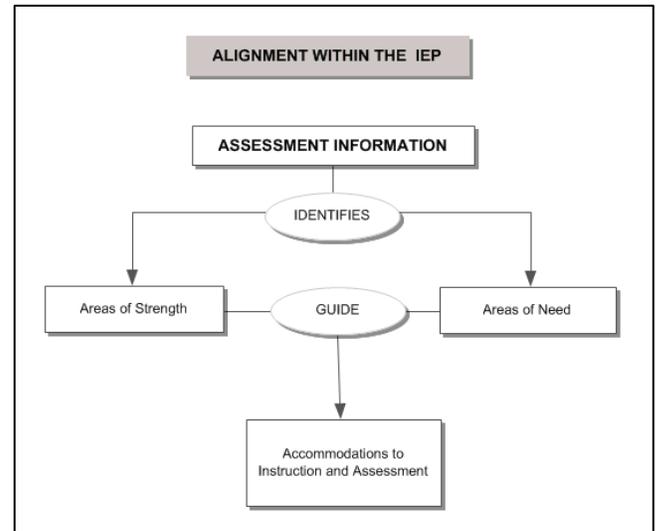

Students in a mathematics class typically demonstrate diversity in the ways they learn best. It is important, therefore, that students have opportunities to learn in a variety of ways – individually, cooperatively, independently, with teacher direction, through hands-on experience, through examples followed by practice. In mathematics, students are required to learn concepts, procedures, and processes and to acquire skills, and they become competent in these various areas with the aid of the instructional and learning strategies best suited to the particular type of learning. The approaches and strategies used in the classroom to help students meet the expectations of this curriculum will vary according to the object of the learning and the needs of the students. [8]

Even at the secondary level, manipulatives are necessary tools for supporting the effective learning of mathematics. These concrete learning tools invite students to explore and represent abstract mathematical ideas in varied, concrete, tactile, and visually rich ways. Manipulatives are also a valuable aid to teachers. By analyzing students' concrete representations of mathematical concepts and listening carefully to their reasoning, teachers can gain useful insights into students' thinking and provide supports to help enhance their thinking. [8]

An Individual Education Plan (IEP) is:

- a written plan describing the special education program and/or services required by a particular student, based on a thorough assessment of the student's strengths and needs; that is, the strengths and needs that affect the student's ability to learn and to demonstrate learning; [9]

- a record of the particular accommodations needed to help the student achieve his or her learning expectations, given the student's identified learning strengths and needs; [9]

- an alignment of IEP content including assessment information, areas of strength and needs, and accommodations;

- an accountability tool for the student, the student's parents, and everyone who has responsibilities under the plan for helping the student meet the stated goals and learning expectations as the student progresses through the Ontario curriculum. [9]

ALIGNMENT WITHIN THE IEP

ASSESSMENT INFORMATION

IDENTIFIES

Areas of Strength — GUIDE — Areas of Need

Accommodations to Instruction and Assessment

Accommodations:

- are specific teaching and assessment strategies, human supports, and/or individualized equipment required to enable a student to learn and to demonstrate learning [10]

- do not alter provincial curriculum expectations [10]

- once listed in the IEP, MUST be made readily available to the student. [10]

When Implementing Accommodations It Is Important To Remember:

- Fairness is not sameness.   Treating all students exactly the same means that children who need accommodations in order to succeed will be disadvantaged. [11]

- Accommodations need to be highly individualized in response to a student learning profile.   In order to be effective, instructional and assessment accommodations must be in direct response to student strengths and needs.

- Multiple accommodations may be suitable in addressing an area of need (e.g. base ten blocks, an interactive electronic rekenrek tool [www.mathies.ca], a multiplication table, or a calculator are possible accommodations for a student who cannot remember multiplication facts).

A SAMPLE Individual Education Plan

Student ID:		School:	
Date of Birth:		Principal:	
Grade:	6	School Year:	2011-12

DRAFT
as of 11 Oct 2012

Reason for IEP:	Student identified as exceptional by IPRC

Most Recent IPRC:	20 Apr 2011
Identification:	**Learning Disability**
IPRC Placement:	Regular Class with Withdrawal
Program:	Core Resource
Start of Program:	06 Sep 2011
IEP completed:	IEP revised: 11 Oct 2012

Assessments

Type	Information Source	Date	Summary Statement
Psychological	Dr. M. Droste	19 Mar 2008	Student is diagnosed with a Learning Disability. Broad test scores are in the average range. Student demonstrates strength in verbal comprehension and good short term and working memory. She has significant difficulty with perceptual reasoning and below average processing speed. Executive functioning is a challenge.

Student Profile

Areas of Strength:
- auditory learner
- verbal comprehension
- oral expression
- listening comprehension
- short term and working memory
- motivated student
- works well in groups

Areas of Need:
- decoding
- reading comprehension
- organization of written ideas
- perceptual reasoning: understanding part-whole relationships, patterns and sequences
- interpreting visual information (e.g. diagrams, graphs, charts)
- processing speed
- becomes overwhelmed by volume of information
- executive functioning: planning, initiating tasks and organization

Program Summary

This IEP applies to the following programs, subjects or skill areas:

Accommodate only
- Language
- Mathematics
- Social Studies
- Science

Accommodations

Instructional Accommodations:
- always pair verbal with visuals
- ensure access to learning tools (e.g. manipulatives, interactive electronic learning tools) and technology (e.g. Kurzweil, Inspiration, Dragon Speaking)
- make explicit parts and whole, and how they connect
- provide extra time for processing information
- provide opportunities to process information and ideas verbally
- present information in manageable chunks while maintaining the cognitive demand of the task
- provide organizational supports: e.g. check list, graphic organizers

Assessment Accommodations:
- allow for oral explanations of written responses
- provide access to learning tools (e.g. manipulatives) and technology (e.g. Kurzweil, Inspiration, Dragon Speaking)
- provide additional time to complete assessment tasks
- verbatim reading of instructions for all assessments including EQAO

Alignment of IEP Content

Tests of Cognitive Ability

Tests of Cognitive Ability are standardized tests that measure a person's aptitude or potential for learning.

Tests are norm referenced. The individual is scored in comparison to individuals of the same age. When plotted on a graph, the scores of the group will form a "normal curve".

Scores are reported in percentiles. If a person scores at the 50th percentile, 50% of their peers would score higher and 49% would score lower.
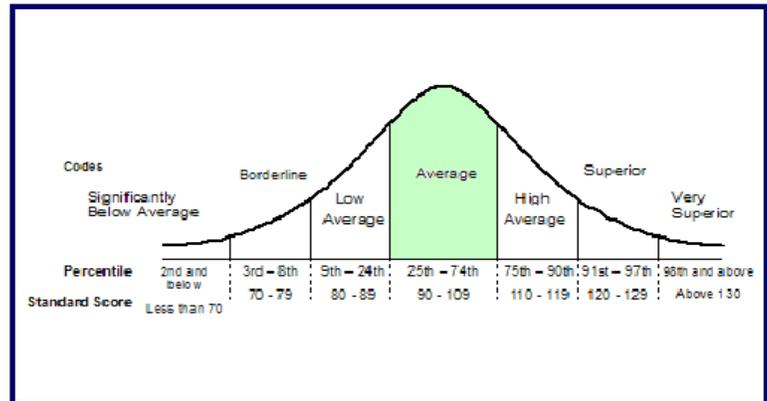
Scores ranging from the 25th to 74th percentile, are considered to reflect average cognitive ability.

Assessments of cognitive abilities include broad areas of testing: Verbal Comprehension, Perceptual Reasoning, Memory, Processing Speed, and Visual Motor Integration. In this document these areas are referred to as the *Cognitive Domains / Processes.*

Executive Functioning is included in this document because of its broad impact on learning and demonstration of learning. Individuals with a learning disability often experience challenges with executive functioning.

Students with Learning Disabilities

On tests of cognitive ability, students with learning disabilities:

- show average to above average cognitive ability;

- struggle in one or more areas of cognitive processing, affecting their ability to perceive or process verbal or non-verbal information in an effective and accurate manner;

- demonstrate considerable differences between cognitive strengths and needs;

- each have a unique profile of strengths and needs
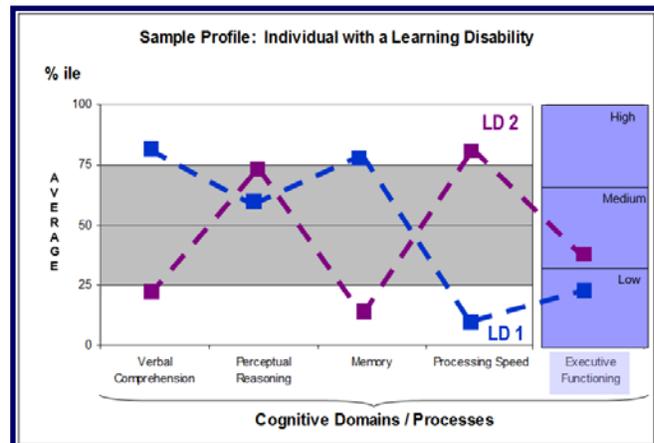
These exceptional learners often experience discrepancies between their potential for learning and their academic achievement. With carefully selected accommodations that leverage student strengths and support areas of need, these students can achieve their potential in mathematics.

Format of Document

This document is structured around the cognitive domains / processes. It identifies the student actions in mathematics that align to each. Instructional and assessment accommodations that leverage student strengths and support needs are included.
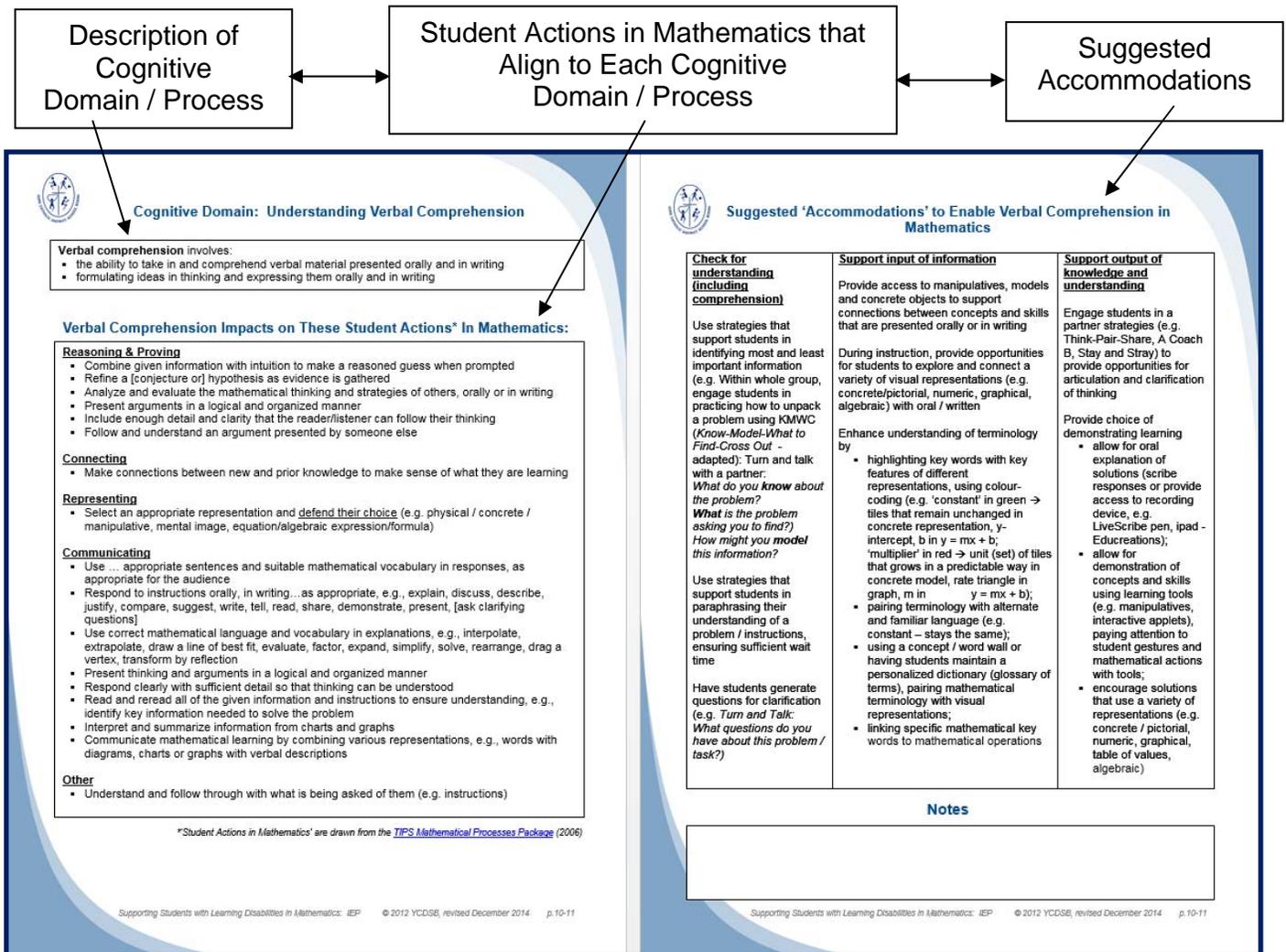
As mathematics and special education teachers use this document when collaborating, they will deepen their understanding of individual student profiles, and select effective accommodations to support student learning in mathematics.

| Description of Cognitive Domain / Process | Student Actions in Mathematics that Align to Each Cognitive Domain / Process | Suggested Accommodations |

Cognitive Domain: Understanding Verbal Comprehension

Verbal comprehension involves:
- the ability to take in and comprehend verbal material presented orally and in writing
- formulating ideas in thinking and expressing them orally and in writing

Verbal Comprehension Impacts on These Student Actions* In Mathematics:

Reasoning & Proving
- Combine given information with intuition to make a reasoned guess when prompted
- Refine a [conjecture or] hypothesis as evidence is gathered
- Analyze and evaluate the mathematical thinking and strategies of others, orally or in writing
- Present arguments in a logical and organized manner
- Include enough detail and clarity that the reader/listener can follow their thinking
- Follow and understand an argument presented by someone else

Connecting
- Make connections between new and prior knowledge to make sense of what they are learning

Representing
- Select an appropriate representation and defend their choice (e.g. physical / concrete / manipulative, mental image, equation/algebraic expression/formula)

Communicating
- Use ... appropriate sentences and suitable mathematical vocabulary in responses, as appropriate for the audience
- Respond to instructions orally, in writing…as appropriate, e.g., explain, discuss, describe, justify, compare, suggest, write, tell, read, share, demonstrate, present, [ask clarifying questions]
- Use correct mathematical language and vocabulary in explanations, e.g., interpolate, extrapolate, draw a line of best fit, evaluate, factor, expand, simplify, solve, rearrange, drag a vertex, transform by reflection
- Present thinking and arguments in a logical and organized manner
- Respond clearly with sufficient detail so that thinking can be understood
- Read and reread all of the given information and instructions to ensure understanding, e.g., identify key information needed to solve the problem
- Interpret and summarize information from charts and graphs
- Communicate mathematical learning by combining various representations, e.g., words with diagrams, charts or graphs with verbal descriptions

Other
- Understand and follow through with what is being asked of them (e.g. instructions)

*Student Actions in Mathematics' are drawn from the TIPS Mathematical Processes Package (2006)

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Suggested 'Accommodations' to Enable Verbal Comprehension in Mathematics

Check for understanding (including comprehension)	Support input of information	Support output of knowledge and understanding
Use strategies that support students in identifying most and least important information (e.g. Within whole group, engage students in practicing how to unpack a problem using KMWC (*Know-Model-What to Find-Cross Out* - adapted): Turn and talk with a partner: *What do you know about the problem? What is the problem asking you to find?) How might you model this information?* Use strategies that support students in paraphrasing their understanding of a problem / instructions, ensuring sufficient wait time Have students generate questions for clarification (e.g. *Turn and Talk: What questions do you have about this problem / task?*)	Provide access to manipulatives, models and concrete objects to support connections between concepts and skills that are presented orally or in writing During instruction, provide opportunities for students to explore and connect a variety of visual representations (e.g. concrete/pictorial, numeric, graphical, algebraic) with oral / written Enhance understanding of terminology by - highlighting key words with key features of different representations, using colour-coding (e.g. 'constant' in green → tiles that remain unchanged in concrete representation, y-intercept, b in y = mx + b; 'multiplier' in red → unit (set) of tiles that grows in a predictable way in concrete model, rate triangle in graph, m in y = mx + b); - pairing terminology with alternate and familiar language (e.g. constant – stays the same); - using a concept / word wall or having students maintain a personalized dictionary (glossary of terms), pairing mathematical terminology with visual representations; - linking specific mathematical key words to mathematical operations	Engage students in a partner strategies (e.g. Think-Pair-Share, A Coach B, Stay and Stray) to provide opportunities for articulation and clarification of thinking Provide choice of demonstrating learning - allow for oral explanation of solutions (scribe responses or provide access to recording device, e.g. LiveScribe pen, ipad - Educreations); - allow for demonstration of concepts and skills using learning tools (e.g. manipulatives, interactive applets), paying attention to student gestures and mathematical actions with tools; - encourage solutions that use a variety of representations (e.g. concrete / pictorial, numeric, graphical, table of values, algebraic)

Notes

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Cognitive Domain: Understanding Verbal Comprehension

Verbal comprehension involves:
- the ability to take in and comprehend verbal (language-based) information (presented orally and in writing)
- formulating ideas in thinking, and expressing them (orally and in writing)

Verbal Comprehension Impacts on These Student Actions* In Mathematics:

Reasoning & Proving
- Combine given information with intuition to make a reasoned guess when prompted
- Refine a [conjecture or] hypothesis as evidence is gathered
- Analyze and evaluate the mathematical thinking and strategies of others, orally or in writing
- Present arguments in a logical and organized manner
- Include enough detail and clarity that the reader/listener can follow their thinking
- Follow and understand an argument presented by someone else

Connecting
- Make connections between new and prior knowledge to make sense of what they are learning

Representing
- Select an appropriate representation and <u>defend their choice</u> (e.g. physical / concrete / manipulative, mental image, equation/algebraic expression/formula)

Communicating
- Use … appropriate sentences and suitable mathematical vocabulary in responses, as appropriate for the audience
- Respond to instructions orally, in writing…as appropriate, e.g., explain, discuss, describe, justify, compare, suggest, write, tell, read, share, demonstrate, present, [ask clarifying questions]
- Use correct mathematical language and vocabulary in explanations, e.g., interpolate, extrapolate, draw a line of best fit, evaluate, factor, expand, simplify, solve, rearrange, drag a vertex, transform by reflection
- Present thinking and arguments in a logical and organized manner
- Respond clearly with sufficient detail so that thinking can be understood
- Read and reread all of the given information and instructions to ensure understanding, e.g., identify key information needed to solve the problem
- Interpret and <u>summarize</u> information from charts and graphs
- Communicate mathematical learning by combining various representations, e.g., words with diagrams, charts or graphs with verbal descriptions

Other
- Understand and follow through with what is being asked of them (e.g. instructions)

*'Student Actions in Mathematics' are drawn from the <u>TIPS Mathematical Processes Package</u> (2006)

Suggested 'Accommodations' to Enable Verbal Comprehension in Mathematics

Check for understanding (including comprehension)	**Support input of information**	**Support output of knowledge and understanding**
Use strategies that support students in identifying most and least important information (e.g. Within whole group, engage students in unpacking problems using KMWC (*Know-Model-What to Find-Cross Out* - adapted): Turn and talk with a partner: *What do you **know** about the problem? **What** is the problem asking you to find? How might you **model** this information?)* Use strategies that support students in paraphrasing their understanding of a problem / instructions, ensuring sufficient wait time Have students generate clarifying questions (e.g. *Turn and Talk: What questions do you have about this problem / task?*)	Provide access to manipulatives (or interactive electronic tools / supports), models, and concrete objects to support instruction (verbal) and group math talk During instruction, provide opportunities for students to explore and connect a variety of visual representations (e.g. concrete/pictorial, numeric, graphical, algebraic) with oral / written Enhance understanding of terminology by ▪ highlighting key words with key features of different representations, using colour-coding (e.g. 'constant' in green → tiles that remain unchanged in concrete representation, y-intercept, b-value in $y = mx + b$; 'multiplier' in red → unit (set) of tiles that grows in a predictable way in concrete model, rate triangle in graph, m-value in $y = mx + b$); ▪ pairing terminology with alternate and familiar language (e.g. constant – stays the same); ▪ using a concept / word wall or having students maintain a personalized dictionary (glossary of terms), pairing mathematical terminology with visual representations; ▪ linking specific mathematical key words to mathematical operations	Engage students in a partner strategies (e.g. Think-Pair-Share, A Coach B, Stay and Stray) to provide opportunities for articulation and clarification of thinking Provide choice of demonstrating learning ▪ allow for oral explanation of solutions (scribe responses or provide access to recording device, e.g. LiveScribe pen, ipad - Educreations); ▪ allow for demonstration of concepts and skills using learning tools (e.g. manipulatives, interactive electronic tools), paying attention to student gestures and mathematical actions with tools; ▪ encourage solutions that use a variety of representations (e.g. concrete / pictorial, numeric, graphical, table of values, algebraic)

Notes

Cognitive Domain: Understanding Perceptual Reasoning

Perceptual Reasoning involves visual spatial and fluid reasoning:
- the ability to understand visual-spatial information, such as part-whole relations, patterns, and sequences
- the capacity to think logically and solve problems in unfamiliar situations
- the ability to generate visual representations in the mind, and present ideas in a visual format

Perceptual Reasoning Impacts on These Student Actions*
In Mathematics

Problem Solving
- Understand the problem
- Ask thoughtful questions
- Search the Internet for secondary <u>data;</u> check that <u>data</u> being gathered is appropriate to the inquiry…
- [Use]…strategies [that are visual in nature] (e.g. draw a diagram or picture, create a mathematical model, look for a pattern, make a scale drawing, make an organized list)

Reasoning & Proving
- Refine hypothesis as evidence is gathered
- Use models … to infer/conclude; adjust models, as needed
- Reason inductively by considering specific cases and identifying patterns
- Try multiple examples, e.g., make multiple trials using a GSP® sketch; make systematic trials using manipulatives…
- Look for a case that does not work, i.e., a counterexample
- Recognize the characteristics of an acceptable argument/proof

Reflecting
- Search for relevant primary and secondary data
- Check that data being gathered is appropriate to the inquiry
- Verify a solution to a problem by using a different [visual] method

Selecting Tools and Computational Strategies
- Use technology (e.g., graphing calculators, spreadsheets, The Geometer's Sketchpad® , Fathom®, Tinkerplots®) to explore, gather, display, manipulate, and present data in a variety of ways
- Use manipulatives and/or technology to develop understanding of new concepts, for communicating, or for performing certain tasks
- <u>Estimate, using properties of numbers</u>, e.g., a square root from known square numbers
- Select different computational strategies <u>depending on the numbers involved</u>, e.g., 25 × 16, 23 × 16, 19 × 16

Connecting
- Use different models to best convey mathematical information and demonstrate their conceptual understanding of a procedure
- Make connections between different representations, e.g., numeric, graphical, and/or algebraic

Representing
- Select an appropriate representation and defend their choice, such as: physical/concrete/manipulative;, electronically generated, e.g. graphs, dynamic geometry representation; mental image; numerical, e.g. table of values; graphical; scale drawing; diagram; equation/algebraic expression/formula; algorithm/logic model)
- Understand that various representations can be used to appropriately represent the same situation
- Understand that there may be different variations of one representation, e.g., algebraic expressions may be equivalent yet appear different

Communicating
- Interpret and summarize information from charts and graphs
- Use the symbolic language of mathematics correctly, e.g., use = down the left side when simplifying expressions and in between two equal expressions when solving equations; use $\angle$ for naming angles
- Communicate mathematical learning by combining various representations, e.g., words with diagrams, charts or graphs with verbal descriptions

*'Student Actions in Mathematics' are drawn from the <u>TIPS Mathematical Processes Package</u> (2006)

Suggested Accommodations to Enable Perceptual Reasoning in Mathematics

Support visual representations with oral and/or written	Provide access to tools, including concrete materials and models to enable connections between visual and verbal (oral and written)	Support visual access to content	Support understanding of concepts, including part/whole relations, through visualization and gestures, with the support of manipulatives / tools
Pair visual representations with oral and/or written when teaching (e.g. label 'multiplier' in a pattern, label the rate triangle in a graph with 'rate of change') Use a think aloud strategy when presented with a visual representation and required to: • make a conjecture • solve a problem • make inferences, conclusions and justifications • make connections between representations Provide access to Assistive technology (e.g. mathies interactive electronic tools, Math CLIPS, Kurzweil) during lessons and assessments	Use manipulatives (e.g. Base 10 blocks, geoboards, fraction pieces, algebra tiles), models (e.g. 3D solids with nets), and technology (e.g. graphing software, dynamic geometry software, dynamic statistical software, interactive applets) in combination with language Connect abstract mathematical concepts to concrete models and experiences (e.g. find the area – tile the desk with unit tiles; determine the slope – build 3 different staircases using cube-a-links, with slope of 1, 0.5 and 2; find the volume – use metre sticks to build 1 m^3 and think of as many objects that are about this same volume) Provide access to manipulatives and technology to enable communication of math thinking (e.g. mathies interactive electronic tools, Live Scribe pens, Educreations)	Avoid presenting too many visuals and/or excessive text at one time (decrease clutter) Use colour-scaffolding to identify key characteristics / terms / symbols (e.g. draw a rate triangle using green and colour the y-intercept using yellow; use coloured tiles for integers, e.g. red for +1 and blue for -1) Provide visual and tactile experiences with representations (e.g. use ten frames to represent 28; represent and solve an algebraic equation using algebra tiles; trace the graph of an increasing function with your finger)	Make explicit part/whole relations using manipulatives / interactive electronic tools (e.g. count by fractional amounts using relational rods with overlay on the whole…1 one-fourth; 2 one-fourths; 3 one-fourths; 4 one-fourths or 1 whole) Support students in unitizing using mathematical actions with manipulatives (e.g. drag 5 tiles at a time to build a 6 by 5 array while skip counting by 5's to 30) Make explicit mathematical structure / important features in visual representations (e.g position 3 for the linear growing pattern, $T = n \times 4 + 2$, has 3 rows of 4 green tiles, and 2 red tiles; position 10 for the same linear growing pattern has 10 rows of 4 green tiles and 2 red tiles) Engage students in visualization practice, with the support of manipulatives / tools (e.g. close your eyes and visualize a rectangle; wrap a pipe cleaner around the rectangle; pinch the vertices (corners) of the rectangle; unwrap the pipe cleaner and cut the four segments (pieces); group common-sized segments and describe the 'perimeter' (or length around rectangle) in terms of these segments; open your eyes and test with a pipe cleaner) Use gestures to support visualization (e.g. use hand gestures to model transformations, such as reflections, rotations)

Notes

Memory involves:
- working memory, short-term, and long-term memory (visual memory and auditory memory may also be assessed)
- working memory is the ability to hold information in the mind for processing/manipulating

Memory Impacts on These Student Actions* In Mathematics

Problem Solving
- Check that [and retain] data being gathered is appropriate to the inquiry at hand
- [Use and follow through with] some of the following problem solving strategies [that are multi-step in nature] (e.g. work backwards, monitor progress and revise, as necessary)
- Incorporate different strategies <u>over time</u>

Reasoning & Proving
- Make a reasoned guess as to where <u>in the process</u> and/or why an attempted solution failed
- Reason inductively by considering specific cases and identifying patterns
- <u>Follow</u> and understand an argument presented by someone else

Reflecting
- Apply and extend knowledge to new situations
- Examine questions and demonstrate flexibility in choice of strategy <u>based on the nature of the question</u>
- Self-monitor progress while problem solving and revise, as necessary

Selecting Tools and Computational Strategies
- Perform mental calculations, e.g., estimate by substituting rounded values into formulas; add many large numbers, keeping track of rounding error to adjust the total; multiply by rounding and considering place value; apply the distributive property
- Estimate <u>using properties of numbers</u>, e.g., a square root from known square numbers
- …Use a personal set of referents for measurement, e.g., 1 cm is approximately the width of a baby finger
- Select different computational strategies depending on the numbers involved, e.g., 25×16, 23×16, 19×16

Connecting
- Apply a strategy or reference system that draws on previous learning in another context
- Make connections between new and prior knowledge to make sense of what they are learning

Communicating
- …summarize information from charts and graphs, providing appropriate detail, e.g., describe patterns and contrasts
- Read and reread all of the given information and instructions to ensure understanding, e.g., identify key information needed to solve the problem

Other:
- Follow multi-step instructions orally

*'Student Actions in Mathematics' are drawn from the *TIPS Mathematical Processes Package* (2006)

Suggested Accommodations to Enable Memory in Mathematics

Explicitly teach memory strategies	Provide opportunities for making connections	Provide memory supports without modification of curriculum expectations
Use a think aloud strategy to verbalize mathematics ideas / concepts / procedures (e.g. Think-Pair-Share, A Coach B) Use mnemonics (e.g. visualization, compare and contrast, repetition) following learning experiences that develop conceptual understanding of mathematical concepts with related procedures, problem solving strategies, etc. Use rhythm / music / patterns to support recall of concepts and related procedures Convert concepts, models, notes, etc., into abbreviated form or use visual cues / representations to recall information Read, cover, recite / write	Support students in connecting concepts with corresponding terminology to concrete / visual images (e.g. use of ten frame to visualize quantities; use of algebra tiles to distinguish 1 from x from x^2, picture of ski hill to recall concept of slope; concept map / web including visual representations that triggers recall of concepts / procedures) Use real life situations / contexts to present problems or illustrate concepts	Provide access to manipulatives / interactive electronic tools (with post-it notes if needed) or visual representations to support students in keep tracking of several ideas or quantities simultaneously (e.g. 5 leaves per 2 caterpillars → 5 green tiles to 2 red tiles or double number line; part-part-whole template for problems involving addition and subtraction) Make explicit the mathematical structure within part/whole relations to reduce cognitive load (e.g. reorganize 6 piles of 5 tiles (non-subitized) into 6 rows of 5 tiles or 2 sets of 3 rows of 5 tiles (subitized – array); use ten frames for adding and subtracting double-digit numbers; use algebra tiles for factoring a trinomial) In order to demonstrate learning, provide opportunities to draw on strategies and tools (e.g. manipulatives, visual representations, Bansho artefacts) used during instruction to trigger memory of learning experiences Provide access to artefacts of learning and anchor charts to enable independence and support students in remembering • key terminology (concept / word wall) • formula with visual representation • mnemonics (e.g. KMWC) • step by step model (e.g. Poyla's 4-step problem solving process) During the process of studying, have students create a reference sheet (with teacher input) with personalized cues to trigger memory (e.g. a red tile = +1 and a white tile = -1; an array model to recall multiplication of 2-digit by 2-digit numbers)

Notes

Cognitive Processes: Understanding Processing Speed and Visual Motor Integration

Processing Speed and **Visual Motor Integration**:
- Processing speed is the ability to perform simple visual tasks **quickly and accurately**. It may apply to processing of other types of information; this does not imply struggles with understanding and/or reasoning
- Visual motor integration is the ability to coordinate visual input and physical movements to produce written work accurately (e.g., record keeping from the blackboard and transfer to written form)

Processing Speed and Visual Motor Integration Impacts on These Student Actions* In Mathematics

Problem Solving
- Take and record measurements
- Plan (e.g. generate some examples)
- Select and apply a problem-solving strategy

Reasoning and Proving
- Reason inductively by…identifying patterns
- Analyze and evaluate the mathematical thinking and strategies of others, orally or in writing
- Look for a case that does not work, i.e., a counterexample

Reflecting
- Check that data being gathered is appropriate to the inquiry
- Consider the reasonableness of their answer

Selecting Tools and Computational Strategies
- Perform mental calculations, e.g., estimate by substituting rounded values into formulas; add many large numbers, keeping track of rounding error to adjust the total; multiply by rounding and considering place value; apply the distributive property
- Estimate using properties of numbers, e.g., a square root from known square numbers

Connecting
- Make connections between different representations, e.g., numeric, graphical, and/or algebraic

Representing
- Understand that there may be different variations of one representation, e.g., algebraic expressions may be equivalent yet appear different (e.g. e.g. 4(n+1) = 4n+4, $\left(\frac{(x+1)(x+2)}{(x+1)} = \frac{x+2}{x+1} \right)$)

Communicating
- Interpret and summarize information from charts and graphs, providing appropriate detail, e.g., describe patterns and contrasts
- Read and reread all of the given information and instructions to ensure understanding, e.g., identify key information needed to solve the problem
- Communicate mathematical learning by combining various representations, e.g., words with diagrams, charts or graphs with verbal descriptions

Other
- Organize information on a page (spatially)
- Quality, speed and amount of written work
- Record notes from the blackboard that are legible, organized and include accurate mathematical form
- Produce or reproduce a solution that may include a diagram, table of values, equation, graph, etc.

Suggested Accommodations to Enable Processing Speed and Visual Motor Integration in Mathematics

Provide students with tools to assist them with processing of information	Provide students with strategies for processing information (ie. one task, one step at a time)	Provide time for mathematical thinking, and minimize time and fatigue for students
As an alternative to writing, provide access to manipulatives for learning and demonstrating understanding of concepts Use assistive technology (e.g. interactive electronic tools, Math CLIPS, Kurzweil, Inspiration, recording devices such as LiveScribe pen, ipad, Geometer's sketchpad)	Provide a structure that assists students in identifying similarities and differences (e.g. graphic organizer) Cover up all but one line at a time Focus spotlight on (isolate) one aspect of a visual representation when presenting information Support students in developing strategies to chunk (breakdown or group) information or tasks into manageable and meaningful 'units' to support thinking process while preserving the cognition demand of the task (e.g. use KMWC to determine the most/least important information in a problem; using manipulatives to represent a linear growing pattern will support students in describing the pattern in words or with a pattern rule)	Provide students with more time to process information (e.g. wait time, slow down speed when presenting information, turn and talk with a partner, provide a task / problem in advance, use parallel tasks that focus on the same learning goal) Provide access to learning tools (manipulatives / interactive electronic tools) to support think time and enable alternate ways for students to demonstrate learning (e.g. Base 10 blocks to add two-digit numbers, 3D nets to explain how to calculate surface area of a rectangular prism, algebra tiles to add polynomials, Live Scribe Pens or Educreations to capture student think aloud when solving a problem) Reduce time spent copying notes and solutions (e.g. provide copy of notes / solutions in class, post lesson materials on webpage, have students capture their thinking using recording devices) Teach shortcuts / abbreviations (e.g. km for kilometres) once students have understood the concept Reduce the number of homework questions that practice the same concepts and skills Extend timelines, in conjunction with other strategies Reduce test questions that measure the same overall expectations (ie. draw on the specific expectations that best reflect the overall expectation)

Notes

Executive Functioning involves
- mental processes that draw on past experiences in order to successfully complete a task
- planning, organizing, strategizing, focusing attention, self-monitoring, self-regulating, and managing time and space
- cognitive flexibility (e.g. ability to change one's problem solving strategy)

Executive Functioning Impacts on These Student Actions* In Mathematics

Problem Solving
- Understand the problem
- Search the Internet for secondary data
- Check that data being gathered is appropriate to the inquiry at hand
- Select and apply a problem-solving strategy (e.g. create a mathematical model, draw a diagram or picture, make a simpler but similar problem)

Reflecting
- Consider data collected (e.g. search for relevant primary and secondary data)
- Reflect on new skills, concepts, and questions to see how they connect to prior knowledge (e.g. examine questions and demonstrate flexibility in choice of strategy)
- Self-monitor progress while problem solving and revise, as necessary

Selecting Tools and Computational Strategies
- Select and use an appropriate tool appropriately
- Use technology (e.g., graphing calculators, GSP® , Fathom®) to explore, gather, display, manipulate, and present data in a variety of ways

Connecting
- Apply mathematics to contexts outside of mathematics

Representing
- Select an appropriate representation and defend their choice (e.g. physical / concrete / manipulative, graphical organizers (e.g., Venn diagram, T-chart, concept map), equation/algebraic expression/formula)

Communicating
- Present thinking and arguments in a logical and organized manner
- Respond to instructions orally, in writing, and visually, as appropriate, e.g., explain, discuss, describe, justify, compare, suggest, write, tell, read, share, demonstrate, present
- Interpret and summarize information from charts and graphs, providing appropriate detail, e.g., describe patterns and contrasts
- Communicate mathematical learning by combining various representations, e.g., words with diagrams, charts or graphs with verbal descriptions

Other:
- Begin and stay focused on a task,
- Organize notebook, notes, handouts, etc.
- Time management
- Transition between activities / tasks

*'Student Actions in Mathematics' are drawn from the *TIPS Mathematical Processes Package* (2006)*

Suggested Accommodations to Enable Executive Functioning in Mathematics

Use strategies to support planning, focusing attention	Provide students with strategies for organizing, managing and using information (ie. one task, one concept at a time for students)	Use metacognitive strategies (reflect)	Use of assessment strategies
Activate prior knowledge in preparation for (new) learning task. Use strategies to support students in developing understanding of concepts, solving problems, staying on task during a lesson, and demonstrate their learning • provide access to manipulatives and technology (e.g. e-tools, spreadsheet software, dynamic statistical software, dynamic geometry software, Inspiration, Kurzweil, electronic organizer, LiveScribe pen) • use graphic organizers (e.g. KMWC, Venn diagram, concept map) • provide structures (e.g. Polya's 4-step problem solving process) • post class agenda, timelines and lessons in classroom and on webpage • provide checklist of tasks / timelines	Scaffold information / task / learning into manageable and meaningful 'units', while preserving the cognitive demand Provide students with access to learning tools when solving problems / completing math tasks Make explicit connections to real life applications of mathematics Make accessible learning artefacts that are co-created (e.g. class consolidation of mathematical thinking and/or problem solving strategies, concept / word wall) Promote partner strategies Create a predictable classroom routine (e.g. moving from whole class to group work, routines for accessing learning tools)	Use a think aloud strategy when • making a conjecture • solving a problem • making inferences, conclusions and justifications • making connections between representations Model, promote, and integrate strategies that focus on reflection (e.g. Turn and talk, Think-Pair-Share, review of LiveScribe pen recordings for further annotation / refinement) Use prompts that support students in self-reflection (e.g. *How does the pattern you built match this pattern rule? How do you know your answer is reasonable?*) Model and promote self-monitoring and self-assessment (e.g. checklist created from TIPS Mathematical Processes package – *Role of Student*, adding additional annotations / refinements to a previous LiveScribe pen recording based on teacher feedback and/or new learning)	Provide exemplars for tasks to support students in understanding assessment task structure / format Provide rubric / marking scheme to students to guide task completion and enable self-assessment of mathematical thinking Provide assessment due dates in advance Use frequent checks and feedback to support organization, task completion Provide options / choices for students to demonstrate learning (e.g demonstration using manipulatives, presentation, written product)

Notes

Glossary

accommodations – refers to the special teaching, assessment strategies and/or individualized equipment required to enable students to learn and demonstrate learning. Accommodations do not alter provincial curriculum expectations.

algebraic expression – a collection of symbols, including one or more variables and possibly numbers and operation symbols. For examples *3x + 6, x, 5x, 21 – 2w* are all algebraic expressions. [12]

algorithm – a systematic procedure for carrying out a computation. [13]

BEDMAS – a mnemonic to recall order of operations
  B – brackets
  E – exponents
  D – division
  M – multiplication } In the order they appear
  A – addition
  S – subtraction } In the order they appear

conjecture – a test or prediction based on limited evidence [14]

concept [mathematical]– a connection of mathematical ideas that provides a deep understanding of mathematics; students develop their understanding of mathematical concepts through rich problem-solving experiences [15]

distributive property – the property that allows a number in a multiplication expression to be decomposed into two or more numbers; for example $51 \times 12 = 51 \times 10 + 51 \times 2$; more formally, the distributive property holds that, for three numbers, a, b, and c, $a \times (b + c) = ab + ac$ and $a \times (b - c) = ab - ac$ [16]

exemplar – a model that serves as an ideal

Fathom® - a Ministry licensed dynamic data software program for exploring statistics

GSP® - Geometer's Sketchpad is a Ministry licensed interactive geometry software program for exploring mathematics

KMWC – a graphic organizer that supports students in identifying most/least important information in a problem
  K – What facts do I *KNOW* from the information in the problem?
  M – Can I *MODEL* the situation with a picture or manipulatives?
  W – *WHAT* does the problem ask me to find?
  C – *CROSS OUT* any facts that are not needed. [17]

Kurzweil – an assistive technology, text to speech, learning tool that enables access to curriculum.

Glossary

graphic organizer – a visual that assists students in organizing ideas and making connections

like terms – terms whose powers, ie. variables and their exponents, are the same (e.g. $2x^2$ and $-3x^2$; 4y and y)

manipulatives – objects that students handle and use in constructing or demonstrating their understanding of mathematical concepts and skills; some examples include base ten blocks, connecting cubes, geoboards, pattern blocks, fraction circles, algebra tiles [18]

math talk [learning community] – a community in which individuals assist one another's learning of mathematics by engaging in meaningful mathematical discourse [19]

mathematical model – a mathematical description (e.g., a diagram, a graph, a table of values, an equation, a formula, a physical model, a computer model) of a situation [20]

metacognition – an awareness of one's own thinking process

meta-cognitive strategies – strategies that help an individual understand the way they are processing information (i.e. questioning, visualizing, and synthesizing information)

monomial – an algebraic expression with one term (for example, $5x^2$) [21]

mnemonics – a device, such as a formula or rhyme, used as an aid to remember

overall expectations – the knowledge and skills students are expected to demonstrate by the end of a course.

perceptual activities – activities that pertain to the way an individual processes sensory input such as sounds, smells, sights, colours, shapes, etc.

Poyla's 4-step problem solving process – involves
1. understanding the problem
2. devising a plan
3. carrying out the plan
4. and looking back. [22]

Procedures [mathematical] – the operations, mechanics, algorithms, and calculations used to solve problems [23]

rate triangle – a triangle that visually represents the rate of change on a graph; in the rate triangle, the distance up is called the rise (change in *y*) and the distance across is called the run (change in *x*); (add diagram)

rational expression – the quotient of two polynomials, [where the denominator is not equal to 0]; for example, $\dfrac{x+1}{x^2+5x+6}$

Glossary

representing – create a variety of representations of mathematical ideas (e.g., by using physical models, pictures, numbers, variables, diagrams, graphs, onscreen dynamic representations), connect and compare representations, and select and apply the appropriate representations to solve problems. [24]

scribe – an "official writer" for a student who struggles with output of knowledge and understanding in a written form; this is an accommodation as the scribe is the "hands" of the person who has a learning disability that affects writing

self-regulation – one's capacity to manage and/or alter one's behaviour in accordance to some standards, ideals or goals.

self-talk – a loud or silent talk to one self, that can be used to encourage, motivate, or regulate one's behaviour.

slope – a measure of the steepness of a line, calculated as the ratio of the rise (vertical change between two points) to the run (horizontal change between the same two points) [25]

spatial relationships – the placement of objects in space and/or the relationship between them; e.g. the ability to perceive distance between two objects as well as between one's self and an object.

subitize – to perceive the number or (a group of items) at a glance and without counting

symbolic language – involves the use of symbols (letters, numbers, mathematical symbols) to communicate meaning.

table of values – a table used to record the coordinates of points in a relation (example here) [26]

think aloud strategy – saying out loud what you are thinking about when reading, solving math problems, or responding to questions posed

think-pair-share – students individually consider an issue or problem and then discuss their ideas with a partner [27]

timed retell – pair students, facing each other; direct all partner A students to tell what they know about the topic for 30 seconds; direct partner B to retell the talk for about 30 seconds then to respond (e.g., add additional information); direct partner A to retell what partner B said [28]

visual-motor integration – the ability to coordinate the activities involving eyes and hands such as copying notes, writing, and drawing

visual tracking – the ability to follow something with one's eyes and/or the ability to track one's eyes from left to right; visual tracking is critical to reading fluently with ease

word wall – a wall, chalkboard or bulletin board listing key words that will appear often in a new unit of study, printed on card stock and taped or pinned to the wall/board; the word wall is usually organized alphabetically [29]

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[26] Ontario Mathematics Curriculum, Grades 9&10: Mathematics (Revised, 2005), p. 64

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[28] Think Literacy: Mathematics, Grades 7-9, p. 101

[29] Think Literacy: Mathematics, Grades 7-9, p. 22